

2.3 Extension--Truth tables

Conditional statements can be written using symbolic notation.

If p, then q. p—hypothesis q—conclusion

$p \rightarrow q$ "if p, then q" or "p implies q"

Symbolic Notation

Conditional $p \rightarrow q$

Converse $q \rightarrow p$

Inverse $\sim p \rightarrow \sim q$

Contrapositive $\sim q \rightarrow \sim p$

Biconditional $p \leftrightarrow q$

The truth value of a statement is either true (T) or false (F).

A truth table shows the conditions when a conditional statement is true. It is only false when a true hypothesis produces a false conclusion.

Truth Table for Conditional

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

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Specific Example:

If it is raining, then you have an umbrella.

It's raining and you don't have an umbrella.

It's not raining and you don't have an umbrella.

It's not raining and you have an umbrella.

p	q	$p \rightarrow q$	$q \rightarrow p$	$\sim p \rightarrow \sim q$	$\sim q \rightarrow \sim p$
T	T	T	T	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	F	F	T

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Examples:

Make a truth table for the logical statement.

 $p \rightarrow \sim q$

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| p | q | $\sim q$ | $p \rightarrow \sim q$ |
|---|---|----------|------------------------|
| T | T | F        | F                      |
| T | F | T        | T                      |
| F | T | F        | T                      |
| F | F | T        | T                      |

 $\sim(p \rightarrow q)$ 

| p | q | $(p \rightarrow q)$ | $\sim(p \rightarrow q)$ |
|---|---|---------------------|-------------------------|
| T | T | T                   | F                       |
| T | F | F                   | T                       |
| F | T | T                   | F                       |
| F | F | T                   | F                       |

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Truth tables can also be made for conjunctions (and) and disjunctions (or).

"p and q" is true only when both p and q are true (symbolic  $p \wedge q$ )"p or q" is false only when both p and q are false (symbolic  $p \vee q$ )Only T  
Both True

| Conjunction |   |              |
|-------------|---|--------------|
| p           | q | $p \wedge q$ |
| T           | T | T            |
| T           | F | F            |
| F           | T | F            |
| F           | F | F            |

Only F when Both F

| Disjunction |   |            |
|-------------|---|------------|
| p           | q | $p \vee q$ |
| T           | T | T          |
| T           | F | T          |
| F           | T | T          |
| F           | F | F          |

Ex:  $\sim p \vee q$ 

| p | q | $\sim p$ | $\sim p \vee q$ |
|---|---|----------|-----------------|
| T | T | F        | T               |
| T | F | F        | F               |
| F | T | T        | T               |
| F | F | T        | T               |

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ex:  $(p \vee q) \wedge \sim r$ 

| p | q | r | $p \vee q$ | $\sim r$ |   |
|---|---|---|------------|----------|---|
| T | T | T | T          | F        | F |
| T | F | T | T          | F        | F |
| T | T | F | T          | T        | T |
| T | F | F | T          | T        | T |
| F | T | T | T          | F        | F |
| F | F | T | F          | F        | F |
| F | T | F | T          | T        | T |
| F | F | F | F          | T        | F |

HW

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1. **WRITING** Describe how to use symbolic notation to represent the contrapositive of a conditional statement.

**WRITING STATEMENTS** Use  $p$  and  $q$  to write the symbolic statement in words.

$p$ : Polygon  $ABCDE$  is equiangular and equilateral.

$q$ : Polygon  $ABCDE$  is a regular polygon.

2.  $p \rightarrow q$

3.  $\sim p$

**MAKING TRUTH TABLES** Make a truth table for the logical statement.

7.  $\sim p \rightarrow q$

9.  $\sim (q \rightarrow p)$

4. **LOGICAL EQUIVALENCE** The truth table shows that the conjunction " $p$  and  $q$ " is true only when  $p$  and  $q$  are both true. It also shows that the disjunction " $p$  or  $q$ " is false only when  $p$  and  $q$  are both false.

- Make a truth table for  $\sim (p \text{ or } q)$ .
- Make a truth table for  $(\sim p \text{ and } \sim q)$ .
- Show that  $\sim (p \text{ or } q)$  and  $(\sim p \text{ and } \sim q)$  are logically equivalent.

| $p$ | $q$ | Conjunction<br>$p \text{ and } q$ | Disjunction<br>$p \text{ or } q$ |
|-----|-----|-----------------------------------|----------------------------------|
| T   | T   | T                                 | T                                |
| T   | F   | F                                 | T                                |
| F   | T   | F                                 | T                                |
| F   | F   | F                                 | F                                |

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